# UNIT IV FET Biasing

# **INTRODUCTION**

The biasing levels for a silicon transistor configuration can be obtained using the characteristic equations  $V_{BE} = 0.7 \text{ V}$ ,  $I_C = \beta I_B$ , and  $I_C \cong I_E$ . The linkage between input and output variables is provided by  $\beta$ , which is assumed to be fixed in magnitude for the analysis to be performed. The fact that beta is a constant stablishes a *linear* relationship between  $I_C$  and  $I_B$ . Doubling the value of  $I_B$  will double the level of  $I_C$ , and so on.

For the field-effect transistor, the relationship between input and output quantities is *nonlinear* due to the squared term in Shockley's equation. Linear relationships result in straight lines when plotted on a graph of one variable versus the other, while nonlinear functions result in curves as obtained for the transfer characteristics of a JFET. The nonlinear relationship between  $I_D$  and  $V_{GS}$  can complicate the mathematical approach to the dc analysis of FET configurations. A graphical approach may limit solutions to tenths-place accuracy, but it is a quicker method for most FET amplifiers. Since the graphical approach is in general the most popular, the analysis of this chapter will have a graphical orientation rather than direct mathematical techniques.

Another distinct difference between the analysis of BJT and FET transistors is that the input controlling variable for a BJT transistor is a current level, while for the FET a voltage is the controlling variable. In both cases, however, the controlled variable on the output side is a current level that also defines the important voltage levels of the output circuit.

The general relationships that can be applied to the dc analysis of all FET amplifiers are

$$I_G \cong 0$$
 A (1)

and

$$I_D = I_S \tag{2}$$

For JFETS and depletion-type MOSFETs, Shockley's equation is applied to relate the input and output quantities:

$$I_D = I_{DSS} \left(\frac{1 - V_{GS}}{V_P}\right)^2 \tag{3}$$

For enhancement-type MOSFETs, the following equation is applicable:

$$I_D = k(V_{GS} - V_T)^2 \tag{4}$$

It is particularly important to realize that all of the equations above are for the *device only!* They do not change with each network configuration so long as the device is in the active region. The network simply defines the level of current and voltage associated with the operating point through its own set of equations. In reality, the dc solution of BJT and FET networks is the solution of simultaneous equations established by the device and network. The solution can be determined using a mathematical or graphical approach—a fact to be demonstrated by the first few networks to be analyzed. However, as noted earlier, the graphical approach is the most popular for FET networks and is employed in this book.

The first few sections of this chapter are limited to JFETs and the graphical approach to analysis. The depletion-type MOSFET will then be examined with its increased range of operating points, followed by the enhancement-type MOSFET. Finally, problems of a design nature are investigated to fully test the concepts and procedures introduced in the chapter.

# **I.**FIXED-BIAS CONFIGURATION

e simplest of biasing arrangements for the *n*-channel JFET appears in Fig. .1. Referred to as the fixed-bias configuration, it is one of the few FET configurations that can be solved just as directly using either a mathematical or graphical approach. Both methods are included in this section to demonstrate the difference between the twophilosophies and also to establish the fact that the same solution can be obtained us-ing either method.

The configuration of Fig. 6.1 includes the ac levels  $\ddot{O}_i$  and  $V_o$  and the coupling capacitors ( $C_1$  and  $C_2$ ). Recall that the coupling capacitors are "open circuits" for the dc analysis and low impedances (essentially short circuits) for the ac analysis. The resistor  $R_G$  is present to ensure that  $V_i$  appears at the input to the FET amplifier for the ac analysis (Chapter 9). For the dc analysis,

$$I_G \cong 0 \text{ A}$$
$$V_{R_G} = I_G R_G = (0 \text{ A}) R_G = 0 \text{ V}$$

.

The zero-volt drop across  $R_G$  permits replacing  $R_G$  by a short-circuit equivalent, as appearing in the network of Fig. 2 specifically redrawn for the dc analysis.



The fact that the negative terminal of the battery is connected directly to the de-fined positive potential of  $V_{GS}$  clearly reveals that the polarity of  $V_{GS}$  is directly op-posite to that of  $V_{GG}$ . Applying Kirchhoff's voltage law in the clockwise direction of the indicated loop of Fig. .2 will result in

$$-V_{GG} - V_{GS} = 0$$

$$V_{GS} = -V_{GG}$$
(5)

and

Since  $V_{GG}$  is a fixed dc supply, the voltage  $V_{GS}$  is fixed in magnitude, resulting in notation "fixed-bias configuration."

The resulting level of drain current  $I_D^{\text{the}}$  now controlled by Shockley's equation:

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

Since  $V_{GS}$  is a fixed quantity for this configuration, its magnitude and sign can simply be substituted into Shockley's equation and the resulting level of  $I_D$  calculated. This is one of the few instances in which a mathematical solution to a FET configuration is quite direct.

A graphical analysis would require a plot of Shockley's equation as shown in Fig. 6.3. Recall that choosing  $V_{GS} = V_P/2$  will result in a drain current of  $I_{DSS}/4$  when plotting the equation. For the analysis of this chapter, the three points defined by  $I_{DSS}$ ,  $V_P$ , and the intersection just described will be sufficient for plotting the curve.



In Fig..4,the fixed level of  $V_{GS}$  has been superimposed as a vertical line at  $V_{GS} = -V_{GG}$ . At any point on the vertical line, the level of  $V_{GS}$  is  $-V_{GG}$ —the level of  $I_D$  must simply be determined on this vertical line. The point where the two curves



intersect is the common solution to the configuration-commonly referred to as the quiescent or operating point. The subscript Q will be applied to drain current and gate-to-source voltage to identify their levels at the Q-point. Note in Fig. 6.4 that the quiescent level of  $I_D$  is determined by drawing a horizontal line from the Q-point to the vertical  $I_D$  axis as shown in Fig. 4. It is important to realize that once the net-work of Fig. 1 is constructed and operating, the dc levels of  $I_D$ and  $V_{GS}$  that will be measured by the meters of Fig. 5 are the quiescent values defined by Fig.4.



qui-escent values of ID and  $V_{GS}$ .

(9)

The drain-to-source voltage of the output section can be determined by applying Kirchhoff's voltage law as follows:

$$+V_{DS} + I_D R_D - V_{DD} = 0$$
ad
$$V_{DS} = V_{DD} - I_D R_D$$
(6)

Recall that single-subscript voltages refer to the voltage at a point with ground. For the configuration of Fig. 6.2,

$$V_S = 0 \text{ V}$$
(7)

Using double-subscript notation:

$$V_{DS} = V_D - V_S$$

$$V_D = V_{DS} + V_S = V_{DS} + 0 \text{ V}$$

$$V_D = V_{DS}$$

$$V_{GS} = V_G - V_S$$
(8)

or and

ar

In addition,

 $V_G = V_{GS} + V_S = V_{GS} + 0 \text{ V}$ 

and

The fact that  $V_D = V_{DS}$  and  $V_G = V_{GS}$  is fairly obvious from the fact that  $V_S =$ 0 V, but the derivations above were included to emphasize the relationship that exists between double-subscript and single-subscript notation. Since the configuration requires two dc supplies, its use is limited and will not be included in the forthcoming list of the most common FET configurations.

 $V_G = V_{GS}$ 

or

EXAMPLE .1

Determine the following for the network of Fig..6.



- (b)  $I_{DQ}$ . (c)  $V_{DS}$ .
- (d)  $V_D$ .
- (e)  $V_G$ . (f)  $V_S$ .



916 V



#### Mathematical Approach:

(a) 
$$V_{GS_Q} = -V_{GG} = -2 \mathbf{V}$$
  
(b)  $I_{D_Q} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 = 10 \text{ mA} \left(1 - \frac{-2 \text{ V}}{-8 \text{ V}}\right)^2$   
 $= 10 \text{ mA}(1 - 0.25)^2 = 10 \text{ mA}(0.75)^2 = 10 \text{ mA}(0.5625)$   
 $= 5.625 \text{ mA}$   
(c)  $V_{DS} = V_{DD} - I_D R_D = 16 \text{ V} - (5.625 \text{ mA})(2 \text{ k}\Omega)$   
 $= 16 \text{ V} - 11.25 \text{ V} = 4.75 \text{ V}$   
(d)  $V_D = V_{DS} = 4.75 \text{ V}$   
(e)  $V_G = V_{GS} = -2 \text{ V}$ 

(f) 
$$V_S = \mathbf{0} \mathbf{V}$$

#### **Graphical Approach:**

The resulting Shockley curve and the vertical line at  $V_{GS} = -2$  V are provided in Fig. 7. It is certainly difficult to read beyond the second place without significantly in- $I_D$  (mA)





Figure.6

creasing the size of the figure, but a solution of 5.6 mA from the graph of Fig..7 is quite acceptable. Therefore, for part (a),

 $V_{GS_Q} = -V_{GG} = -2 \mathbf{V}$ (b)  $I_{D_Q} = 5.6 \text{ mA}$ (c)  $V_{DS} = V_{DD} - I_D R_D = 16 \text{ V} - (5.6 \text{ mA})(2 \text{ k}\Omega)$ = 16 V - 11.2 V = 4.8 V(d)  $V_D = V_{DS} = 4.8 \text{ V}$ (e)  $V_G = V_{GS} = -2 \text{ V}$ (f)  $V_S = 0 \text{ V}$ 

The results clearly confirm the fact that the mathematical and graphical approaches generate solutions that are quite close.

# **II.** SELF-BIAS CONFIGURATION

The self-bias configuration eliminates the need for two dc supplies. The controlling gate-to-source voltage is now determined by the voltage across a resistor  $R_S$  introduced in the source leg of the configuration as shown in Fig. 6.8.



For the dc analysis, the capacitors can again be replaced by "open circuits" and the resistor  $R_G$  replaced by a short-circuit equivalent since  $I_G = 0$  A. The result is the network of Fig. 6.9 for the important dc analysis.

The current through  $R_S$  is the source current  $I_S$ , but  $I_S = I_D$  and

$$V_{R_s} = I_D R_S$$

For the indicated closed loop of Fig. 6.9, we find that

$$-V_{GS} - V_{R_S} = 0$$
and
$$V_{GS} = -V_{R_S}$$
or
$$V_{GS} = -I_D R_S$$
(10)

Note in this case that  $V_{GS}$  is a function of the output current  $I_D$  and not fixed in nitude as occurred for the fixed-bias configuration. mag-



Figure 9DC analysis ofthe self-biasconfiguration.

Equation (10) is defined by the network configuration, and Shockley's equation relates the input and output quantities of the device. Both equations relate the same two variables, permitting either a mathematical or graphical solution.

A mathematical solution could be obtained simply by substituting Eq. (10) into Shockley's equation as shown below:

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$
$$= I_{DSS} \left( 1 - \frac{-I_D R_S}{V_P} \right)^2$$
$$I_D = I_{DSS} \left( 1 + \frac{I_D R_S}{V_P} \right)^2$$

or

By performing the squaring process indicated and rearranging terms, an equation of the following form can be obtained:

$$I_D^2 + K_1 I_D + K_2 = 0$$

The quadratic equation can then be solved for the appropriate solution for  $I_D$ .

The sequence above defines the mathematical approach. The graphical approach requires that we first establish the device transfer characteristics as shown in Fig. 10. Since Eq. (10) defines a straight line on the same graph, let us now identify twopoints on the graph that are on the line and simply draw a straight line between the two points.



**Figure 10** Defining a point on the self-bias line.

The second point for Eq. (10) requires that a level of  $V_{GS}$  or  $I_D$  be chosen and the corresponding level of the other quantity be determined using Eq. (10). The re-sulting levels of  $I_D$  and  $V_{GS}$  will then define another point on the straight line and per-mit an actual drawing of the straight line. Suppose, for example, that we choose alevel of  $I_D$  equal to one-half the saturation level. That is,

$$I_D = \frac{I_{DSS}}{2}$$
$$V_{GS} = -I_D R_S = -\frac{I_{DSS} R_S}{2}$$

The result is a second point for the straight-line plot as shown in Fig. .11. The straight line as defined by Eq. (10) is then drawn and the quiescent point obtained at the in-

then



tersection of the straight-line plot and the device characteristic curve. The quiescent values of  $I_D$  and  $V_{GS}$  can then be determined and used to find the other quantities of interest.

The level of  $V_{DS}$  can be determined by applying Kirchhoff's voltage law to the output circuit, with the result that

 $V_{DS} = V_{DD} - I_D(R_S + R_D)$ 

τ7

$$V_{R_S} + V_{DS} + V_{R_D} - V_{DD} = 0$$
$$V_{DS} = V_{DD} - V_{R_S} - V_{R_D} = V_{DD} - I_S R_S - I_D R_D$$

 $I_D = I_S$ 

but

and

and

In addition:

$$V_S = I_D R_S \tag{12}$$

(11)

$$V_G = 0 \text{ V} \tag{13}$$

and

$$V_D = V_{DS} + V_S = V_{DD} - V_{R_D}$$
(14)



#### **Solution**

(a) The gate-to-source voltage is determined by

$$V_{GS} = -I_D R_S$$

Choosing  $I_D = 4$  mA, we obtain

$$V_{GS} = -(4 \text{ mA})(1 \text{ k}\Omega) = -4 \text{ V}$$

The result is the plot of Fig. 6.13 as defined by the network.



If we happen to choose  $I_D = 8$  mA, the resulting value of  $V_{GS}$  would be -8 V, as shown on the same graph. In either case, the same straight line will result, clearly demonstrating that any appropriate value of  $I_D$  can be chosen as long as the corre- sponding value of  $V_{GS}$  as determined by Eq. (10) is employed. In addition, keep in mind that the value of  $V_{GS}$  could be chosen and the value of  $I_D$  calculated with the same resulting plot.

For Shockley's equation, if we choose  $V_{GS} = V_P/2 = -3$  V, we find that  $I_D = I_{DSS}/4 = 8$  mA/4 = 2 mA, and the plot of Fig. 14 will result, representing the char-acteristics of the device. The solution is obtained by superimposing the network char-acteristics defined by Fig. 13 on the device characteristics of Fig. 6.14 and finding the point of intersection of the two as indicated on Fig. 15. The resulting operating point results in a quiescent value of gate-to-source voltage of



**Figure 14** Sketching the device charac-teristics for the JFET of Fig. 6.12.

**Figure 15** Determining the Q-point for the network of Fig. 6.12.

Self-Bias Configuration

(b) At the quiescent point:

$$I_{DQ} = 2.6 \text{ mA}$$
(c) Eq. (6.11):  $V_{DS} = V_{DD} - I_D(R_S + R_D)$   
 $= 20 \text{ V} - (2.6 \text{ mA})(1 \text{ k}\Omega + 3.3 \text{ k}\Omega)$   
 $= 20 \text{ V} - 11.18 \text{ V}$   
 $= 8.82 \text{ V}$ 
(d) Eq. (6.12):  $V_S = I_D R_S$   
 $= (2.6 \text{ mA})(1 \text{ k}\Omega)$   
 $= 2.6 \text{ V}$ 
(e) Eq. (6.13):  $V_G = 0 \text{ V}$   
(f) Eq. (6.14):  $V_D = V_{DS} + V_S = 8.82 \text{ V} + 2.6 \text{ V} = 11.42 \text{ V}$   
or  $V_D = V_{DD} - I_D R_D = 20 \text{ V} - (2.6 \text{ mA})(3.3 \text{ k}\Omega) = 11.42 \text{ V}$ 

EXAMPLE .3

Find the quiescent point for the network of Fig..12 if: (a)  $R_S = 100 \Omega$ . (b)  $R_S = 10 k\Omega$ .

# **Solution**

Note Fig. 6.16.



(a) With the  $I_D$  scale,

$$I_{Do} \cong 6.4 \text{ mA}$$

From Eq. (6.10),

 $V_{GS_Q} \cong -0.64 \text{ V}$ 

(b) With the  $V_{GS}$  scale,

 $V_{GS_Q} \cong -4.6 \text{ V}$ 

From Eq. (6.10),

 $I_{DO} \cong 0.46 \text{ mA}$ 

In particular, note how lower levels of  $R_S$  bring the load line of the network closer to the  $I_D$  axis while increasing levels of  $R_S$  bring the load line closer to the  $V_{GS}$  axis.

## **EXAMPLE 4**

Determine the following for the common-gate configuration of Fig. 17.



(b)  $I_{DQ}$ .

(c)  $V_{D}^{*}$ .

- (d)  $V_G$ .







#### **Solution**

The grounded gate terminal and the location of the input establish strong similarities with the common-base BJT amplifier. Although different in appearance from the ba-sic structure of Fig..8, the resulting dc network of Fig. 18 has the same basic struc-ture as Fig. .9. The dc analysis can therefore proceed in the same manner as recentexamples.

(a) The transfer characteristics and load line appear in Fig. 19. In this case, the second point for the sketch of the load line was determined by choosing (arbitrarily)

 $I_D = 6$  mA and solving for  $V_{GS}$ . That is,

$$V_{GS} = -I_D R_S = -(6 \text{ mA})(680 \Omega) = -4.08 \text{ V}$$

as shown in Fig. 6.19. The device transfer curve was sketched using

$$I_D = \frac{I_{DSS}}{4} = \frac{12 \text{ mA}}{4} = 3 \text{ mA}$$





Figure 18 Sketching the dc equivalent of the network of Fig. 6.17.

Figure 19 Determining the Q-point for the network of Fig. 6.17.

and the associated value of  $V_{GS}$ :

$$V_{GS} = \frac{V_P}{2} = -\frac{6 \text{ V}}{2} = -3 \text{ V}$$

as shown on Fig. 6.19. Using the resulting quiescent point of Fig. 6.19 results in

$$V_{GSO} \cong -2.6$$
 V

(b) From Fig. 6.19,

 $I_{Do} \cong 3.8 \text{ mA}$ 

(c)  $V_D = V_{DD} - I_D R_D$ = 12 V - (3.8 mA)(1.5 k $\Omega$ ) = 12 V - 5.7 V = **6.3 V** (d)  $V_G = 0$  V (e)  $V_S = I_D R_S = (3.8 \text{ mA})(680 \ \Omega)$ = **2.58 V** (f)  $V_{DS} = V_D - V_S$ = 6.3 V - 2.58 V = **3.72 V** 

# **III** VOLTAGE-DIVIDER BIASING

The voltage-divider bias arrangement applied to BJT transistor amplifiers is also ap-plied to FET amplifiers as demonstrated by Fig.20. The basic construction is ex-actly the same, but the dc analysis of each is quite different.  $I_G = 0$  A for FET am-plifiers, but the magnitude of  $I_B$  for common-emitter BJT amplifiers can affect the dclevels of current and voltage in both the input and output circuits. Recall that  $I_B$  pro-vided the link between input and output circuits for the BJT voltage-divider config-uration while  $V_{GS}$  will do the same for the FET configuration.

The network of Fig. 20 is redrawn as shown in Fig. 21 for the dc analysis. Note that all the capacitors, including the bypass capacitor  $C_S$ , have been replaced by an "open-circuit" equivalent. In addition, the source  $V_{DD}$  was separated into two



**Figure 20** Voltage-divider bias arrangement.



Figure.21 Redrawn network of Fig. 6.20 for dc analysis.

alent sources to permit a further separation of the input and output regions of the network. Since  $I_G = 0$  A, Kirchhoff's current law requires that  $I_{R_1} = I_{R_2}$  and the series equivalent circuit appearing to the left of the figure can be used to find the level of  $V_G$ . The voltage  $V_G$ , equal to the voltage across  $R_2$ , can be found using the voltagedivider rule as follows:

$$V_G = \frac{R_2 V_{DD}}{R_1 + R_2}$$
(15)

Applying Kirchhoff's voltage law in the clockwise direction to the indicated loop of Fig. 6.21 will result in

and

$$V_G - V_{GS} - V_{RS} = 0$$
$$V_{GS} = V_G - V_{RS}$$

Substituting  $V_{R_S} = I_S R_S = I_D R_S$ , we have

$$V_{GS} = V_G - I_D R_S \tag{16}$$

The result is an equation that continues to include the same two variables ap-pearing in Shockley's equation:  $V_{GS}$  and  $I_D$ . The quantities  $V_G$  and  $R_S$  are fixed by the network construction. Equation (16) is still the equation for a straight line, but the origin is no longer a point in the plotting of the line. The procedure for plotting Eq. (16) is not a difficult one and will proceed as follows. Since any straight line requires two points to be defined, let us first use the fact that *anywhere* on the hori-zontal axis of Fig. 22 the current  $I_D = 0$  mA. If we therefore select  $I_D$  to be 0 mA, we are in essence stating that we are somewhere on the horizontal axis. The exact lo- cation can be determined simply by substituting  $I_D = 0$  mA into Eq. (16) and find-ing the resulting value of  $V_{GS}$  as follows:

$$V_{GS} = V_G - I_D R_S$$
  
=  $V_G - (0 \text{ mA}) R_S$   
$$V_{GS} = V_G |_{I_D=0 \text{ mA}}$$
(17)  
sult specifies that whenever we plot Eq. (16) if we choose  $I_D = 0 \text{ mA}$ 

and

The result specifies that whenever we plot Eq. (16), if we choose  $I_D = 0$  mA, the value of  $V_{GS}$  for the plot will be  $V_G$  volts. The point just determined appears in Fig..22.



Figure.22 Sketching the network equation for the voltage-divider configuration.

For the other point, let us now employ the fact that at any point on the vertical axis  $V_{GS} = 0$  V and solve for the resulting value of  $I_D$ :

$$V_{GS} = V_G - I_D R_S$$
  

$$0 V = V_G - I_D R_S$$
  

$$I_D = \frac{V_G}{R_S} \Big|_{V_{GS} = 0 V}$$
(18)

The result specifies that whenever we plot Eq. (66), if  $V_{GS} = 0$  V, the level of  $I_D$  is determined by Eq. (18). This intersection also appears on Fig. 22.

The two points defined above permit the drawing of a straight line to represent Eq. (16). The intersection of the straight line with the transfer curve in the region to the left of the vertical axis will define the operating point and the corresponding levels of  $I_D$  and  $V_{GS}$ .

Since the intersection on the vertical axis is determined by  $I_D = V_G/R_S$  and  $V_G$  is fixed by the input network, increasing values of  $R_S$  will reduce the level of the  $I_D$  in-tersection as shown in Fig. 6.23. It is fairly obvious from Fig..23 that:

Increasing values of  $R_S$  result in lower quiescent values of  $I_D$  and more negative values of  $V_{GS}.$ 



**Figure 23** Effect of  $R_S$  on the resulting *Q*-point.

and

Once the quiescent values of  $I_{DQ}$  and  $V_{GSQ}$  are determined, the remaining network analysis can be performed in the usual manner. That is,

$$V_{DS} = V_{DD} - I_D (R_D + R_S)$$
(19)

$$V_D = V_{DD} - I_D R_D \tag{20}$$

$$V_S = I_D R_S \tag{21}$$

$$I_{R_1} = I_{R_2} = \frac{V_{DD}}{R_1 + R_2}$$
 22)

**EXAMPLE 5** 



#### **Solution**

(a) For the transfer characteristics, if  $I_D = I_{DSS}/4 = 8 \text{ mA}/4 = 2 \text{ mA}$ , then  $V_{GS}$  $= V_P/2 = -4 \text{ V}/2 = -2 \text{ V}$ . The resulting curve representing Shockley's equation ap-pears in Fig. 25. The network equation is defined by

$$V_{G} = \frac{R_{2}V_{DD}}{R_{1} + R_{2}}$$
  
=  $\frac{(270 \text{ k}\Omega)(16 \text{ V})}{2.1 \text{ M}\Omega + 0.27 \text{ M}\Omega}$   
= 1.82 V  
 $V_{GS} = V_{G} - I_{D}R_{S}$ 

and

$$g_{SS} = V_G - I_D R_S$$
$$= 1.82 \text{ V} - I_D (1.5 \text{ k}\Omega)$$

When  $I_D = 0$  mA:





Figure.25 Determining the Q-point for the network .

When  $V_{GS} = 0$  V:

$$I_D = \frac{1.82 \text{ V}}{1.5 \text{ k}\Omega} = 1.21 \text{ mA}$$

The resulting bias line appears on Fig. 6.25 with quiescent values of

$$I_{D_Q} = 2.4 \text{ mA}$$
  
and 
$$V_{GS_Q} = -1.8 \text{ V}$$

(b) 
$$V_D = V_{DD} - I_D R_D$$
  
= 16 V - (2.4 mA)(2.4 k $\Omega$ )  
= **10.24 V**

(c) 
$$V_S = I_D R_S = (2.4 \text{ mA})(1.5 \text{ k}\Omega)$$
  
= **3.6 V**

(d) 
$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
  
= 16 V - (2.4 mA)(2.4 k $\Omega$  + 1.5 k $\Omega$ )  
= **6.64 V**  
or  $V_{DS} = V_D - V_S = 10.24$  V - 3.6 V  
= **6.64 V**

(e) Although seldom requested, the voltage  $V_{DG}$  can easily be determined using

$$V_{DG} = V_D - V_G$$
  
= 10.24 V - 1.82 V  
= **8.42 V**

Although the basic construction of the network in the next example is quite different from the voltage-divider bias arrangement, the resulting equations require a solution very similar to that just described. Note that the network employs a supply at the drain and source.



#### **Solution**

(a) An equation for  $V_{GS}$  in terms of  $I_D$  is obtained by applying Kirchhoff's voltage law to the input section of the network as redrawn in Fig. 27.

 $V_{GS} = V_{SS} - I_S R_S$ 

 $I_S = I_D$ 

 $V_{GS} = V_{SS} - I_D R_S$ 

 $-V_{GS} - I_S R_S + V_{SS} = 0$ 

or but

and

 $V_{GS} = 10 \text{ V} - I_D(1.5 \text{ k}\Omega)$ 



 $V_{GS} = V_{SS} = 10 \text{ V}$ 

m(mA)

9 (I<sub>DSS</sub>)

 $I_{D_{O}} = 6.9 \text{ mA}$ 

9

10

 $V_{88} = 10 \text{ V}$ 

For  $V_{GS} = 0$  V,

and

 $0 = 10 \text{ V} - I_D(1.5 \text{ k}\Omega)$  $I_D = \frac{10 \text{ V}}{1.5 \text{ k}\Omega} = 6.67 \text{ mA}$ 

Q-poin

-3 -2 -1

 $(V_P)$ 

The resulting plot points are identified on Fig. 6.28.

 $G + I_{S}$   $F + I_{S}$ 

(23)

**Figure 27** Determining the network equation for the configu-ration of Fig. 26.

**Figure 28** Determining the *Q*-point for the network

The transfer characteristics are sketched using the plot point established by  $V_{GS} = V_P/2 = -3 \text{ V}/2 = -1.5 \text{ V}$  and  $I_D = I_{DSS}/4 = 9 \text{ mA}/4 = 2.25 \text{ mA}$ , as also appearing on Fig. 28. The resulting operating point establishes the following quiescent levels:

0

 $V_{GS}$ 

2

=-0.35 V

$$I_{D_Q} = 6.9 \text{ mA}$$
$$V_{GS_Q} = -0.35 \text{ V}$$

(b) Applying Kirchhoff's voltage law to the output side of Fig. 26 will result

$$in - V_{SS} + I_S R_S + V_{DS} + I_D R_D - V_{DD} = 0$$

Substituting  $I_S = I_D$  and rearranging gives

$$V_{DS} = V_{DD} + V_{SS} - I_D(R_D + R_S)$$
(24)

which for this example results in

$$V_{DS} = 20 \text{ V} + 10 \text{ V} - (6.9 \text{ mA})(1.8 \text{ k}\Omega + 1.5 \text{ k}\Omega)$$
  
= 30 V - 22.77 V  
= **7.23 V**

(c) 
$$V_D = V_{DD} - I_D R_D$$
  
= 20 V - (6.9 mA)(1.8 k $\Omega$ ) = 20 V - 12.42 V  
= **7.58 V**  
(d)  $V_{DS} = V_D - V_S$   
or  $V_S = V_D - V_{DS}$   
= 7.58 V - 7.23 V  
= **0.35 V**

# **IV.** DEPLETION-TYPE MOSFETs

The similarities in appearance between the transfer curves of JFETs and depletiontype MOSFETs permit a similar analysis of each in the dc domain. The primary difference between the two is the fact that depletion-type MOSFETs permit operating points with positive values of  $V_{GS}$  and levels of  $I_D$  that exceed  $I_{DSS}$ . In fact, for all the configurations discussed thus far, the analysis is the same if the JFET is replaced by a depletion-type MOSFET.

The only undefined part of the analysis is how to plot Shockley's equation for positive values of  $V_{GS}$ . How far into the region of positive values of  $V_{GS}$  and values of  $I_D$  greater than  $I_{DSS}$  does the transfer curve have to extend? For most situations, this required range will be fairly well defined by the MOSFET parameters and the resulting bias line of the network. A few examples will reveal the impact of the change in device on the resulting analysis.



#### **Solution**

(a) For the transfer characteristics, a plot point is defined by  $I_D = I_{DSS}/4 = 6 \text{ mA}/4 = 1.5 \text{ mA}$  and  $V_{GS} = V_P/2 = -3 \text{ V}/2 = -1.5 \text{ V}$ . Considering the level of  $V_P$  and the fact that Shockley's equation defines a curve that rises more rapidly as  $V_{GS}$  becomes more positive, a plot point will be defined at  $V_{GS} = +1 \text{ V}$ . Substituting into Shockley's equation yields

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2$$
  
= 6 mA  $\left( 1 - \frac{+1 V}{-3 V} \right)^2$  = 6 mA $\left( 1 + \frac{1}{3} \right)^2$  = 6 mA(1.778)  
= 10.67 mA

The resulting transfer curve appears in Fig. 6.30. Proceeding as described for JFETs, we have:

Eq. (6.15): 
$$V_G = \frac{10 \text{ M}\Omega(18 \text{ V})}{10 \text{ M}\Omega + 110 \text{ M}\Omega} = 1.5 \text{ V}$$
  
Eq. (6.16):  $V_{GS} = V_G - I_D R_S = 1.5 \text{ V} - I_D (750 \Omega)$ 



Setting  $I_D = 0$  mA results in

$$V_{GS} = V_G = 1.5 \text{ V}$$

Setting  $V_{GS} = 0$  V yields

$$I_D = \frac{V_G}{R_S} = \frac{1.5 \text{ V}}{750 \Omega} = 2 \text{ mA}$$

The plot points and resulting bias line appear in Fig. 6.30. The resulting operating point:

$$I_{DQ} = 3.1 \text{ mA}$$
$$V_{GSQ} = -0.8 \text{ V}$$

(b) Eq. (6.19): 
$$V_{DS} = V_{DD} - I_D(R_D + R_S)$$
  
= 18 V - (3.1 mA)(1.8 k $\Omega$  + 750  $\Omega$ )  
 $\cong$  **10.1 V**

**EXAMPLE 8** 

Repeat Example 7 with  $R_S = 150 \ \Omega$ .

# **Solution**

(a) The plot points are the same for the transfer curve as shown in Fig. 31. For the bias line,

$$V_{GS} = V_G - I_D R_S = 1.5 \text{ V} - I_D (150 \Omega)$$



Setting  $I_D = 0$  mA results in

 $V_{GS} = 1.5 \text{ V}$ 

Setting  $V_{GS} = 0$  V yields

$$I_D = \frac{V_G}{R_S} = \frac{1.5 \text{ V}}{150 \Omega} = 10 \text{ mA}$$

The bias line is included on Fig. 6.31. Note in this case that the quiescent point results in a drain current that exceeds  $I_{DSS}$ , with a positive value for  $V_{GS}$ . The result:

$$I_{DQ} = 7.6 \text{ mA}$$
  
 $V_{GSQ} = +0.35$   
**V**(b) Eq. (19):  $V_{DS} = V_{DD} - I_D(R_D + R_S)$   
 $= 18 \text{ V} - (7.6 \text{ mA})(1.8 \text{ k}\Omega + 150 \Omega)$   
 $= 3.18 \text{ V}$ 

**EXAMPLE 9** 





# **Solution**

(a) The self-bias configuration results in

 $V_{GS} = -I_D R_S$ 

as obtained for the JFET configuration, establishing the fact that  $V_{GS}$  must be less than zero volts. There is therefore no requirement to plot the transfer curve for positive values of  $V_{GS}$ , although it was done on this occasion to complete the transfer characteristics. A plot point for the transfer characteristics for  $V_{GS} < 0$  V is

$$I_D = \frac{I_{DSS}}{4} = \frac{8 \text{ mA}}{4} = 2 \text{ mA}$$
  
 $V_{GS} = \frac{V_P}{2} = \frac{-8 \text{ V}}{2} = -4 \text{ V}$ 

and

and for  $V_{GS} > 0$  V, since  $V_P = -8$  V, we will choose

 $V_{GS} = +2 \text{ V}$ 

and

$$I_D = I_{DSS} \left( 1 - \frac{V_{GS}}{V_P} \right)^2 = 8 \text{ mA} \left( 1 - \frac{+2 \text{ V}}{-8 \text{ V}} \right)^2$$
  
= 12.5 mA

The resulting transfer curve appears in Fig. 6.33. For the network bias line, at  $V_{GS} = 0$  V,  $I_D = 0$  mA. Choosing  $V_{GS} = -6$  V gives

$$I_D = -\frac{V_{GS}}{R_S} = -\frac{-6 \text{ V}}{2.4 \text{ k}\Omega} = 2.5 \text{ mA}$$

The resulting *Q*-point:

$$I_{D_Q} = 1.7 \text{ mA}$$
$$V_{GSQ} = -4.3 \text{ V}$$

(b) 
$$V_D = V_{DD} - I_D R_D$$
  
= 20 V - (1.7 mA)(6.2 k $\Omega$ )  
= **9.46 V**



The example to follow employs a design that can also be applied to JFET transistors. At first impression it appears rather simplistic, but in fact it often causes some confusion when first analyzed due to the special point of operation.

Determine  $V_{DS}$  for the network of Fig. 34.

#### **Solution**

The direct connection between the gate and source terminals requires that

$$V_{GS} = 0 \text{ V}$$

Since  $V_{GS}$  is fixed at 0 V, the drain current must be  $I_{DSS}$  (by definition). In other words,

and

 $V_{GS_Q} = \mathbf{0} \mathbf{V}$  $I_{D_Q} = \mathbf{10} \mathbf{mA}$ 

There is therefore no need to draw the transfer curve and

 $V_D = V_{DD} - I_D R_D = 20 \text{ V} - (10 \text{ mA})(1.5 \text{ k}\Omega)$ = 20 V - 15 V = 5 V

# **V.** ENHANCEMENT-TYPE MOSFETs

The transfer characteristics of the enhancement-type MOSFET are quite different from those encountered for the JFET and depletion-type MOSFETs, resulting in a graphi-cal solution quite different from the preceding sections. First and foremost, recall that for the *n*-channel enhancement-type MOSFET, the drain current is zero for levels of gate-to-source voltage less than the threshold level  $V_{GS(Th)}$ , as shown in Fig. 35. For levels of  $V_{GS}$  greater than  $V_{GS(Th)}$ , the drain current is defined by



**EXAMPLE 10** 

Figure 34 Example.10.



**Figure 6.35** Transfer characteristics of an *n*-channel enhancement-type MOSFET.

$$I_D = k(V_{GS} - V_{GS(\text{Th})})^2$$
(25)

Since specification sheets typically provide the threshold voltage and a level of drain current  $(I_{D(\text{on})})$  and its corresponding level of  $V_{GS(\text{on})}$ , two points are defined immediately as shown in Fig. 35. To complete the curve, the constant *k* of Eq. (25) must be determined from the specification sheet data by substituting into Eq. (25) and solving for *k* as follows:

$$I_{D} = k(V_{GS} - V_{GS(Th)})^{2}$$

$$I_{D(on)} = k(V_{GS(on)} - V_{GS(Th)})^{2}$$

$$k = \frac{I_{D(on)}}{(V_{GS(on)} - V_{GS(Th)})^{2}}$$
(26)

and

Once k is defined, other levels of  $I_D$  can be determined for chosen values of  $V_{GS}$ .

# **Feedback Biasing Arrangement**

A popular biasing arrangement for enhancement-type MOSFETs is provided in Fig. 36. The resistor  $R_G$  brings a suitably large voltage to the gate to drive the MOSFET"on." Since  $I_G = 0$  mA and  $V_{RG} = 0$  V, the dc equivalent network appears as shown in Fig. 37.

A direct connection now exists between drain and gate, resulting in

$$V_D = V_G$$

$$V_{DS} = V_{GS}$$
(27)

and



Figure 36 Feedback biasing arrangement.



Figure 37 DC equivalent of the network of Fig. 36.

For the output circuit,

$$V_{DS} = V_{DD} - I_D R_D$$

which becomes the following after substituting Eq. (6.27):

$$V_{GS} = V_{DD} - I_D R_D \tag{28}$$

The result is an equation that relates the same two variables as Eq. (25), permitting the plot of each on the same set of axes.

Since Eq. (28) is that of a straight line, the same procedure described earlier can be employed to determine the two points that will define the plot on the graph. Sub-stituting  $I_D = 0$  mA into Eq. (28) gives

$$V_{GS} = V_{DD} \big|_{I_D} = 0 \text{ mA}$$
<sup>(29)</sup>

Substituting  $V_{GS} = 0$  V into Eq. (6.28), we have

$$I_D = \frac{V_{DD}}{R_D} \bigg|_{V_{GS} = 0 \text{ V}}$$
(30)

The plots defined by Eqs. (25) and (28) appear in Fig. 38 with the resulting operating point.



**Figure 38** Determining the *Q*-point for the network of Fig. 36.

Determine  $I_{DQ}$  and  $V_{DSQ}$  for the enhancement-type MOSFET of Fig. 39.



# **Solution**

**Plotting the Transfer Curve:** 

Two points are defined immediately as shown in Fig. 6.40. Solving for k:

Eq. (6.26): 
$$k = \frac{I_{D(\text{on})}}{(V_{GS(\text{on})} - V_{GS(\text{Th})})^2}$$
  
=  $\frac{6 \text{ mA}}{(8 \text{ V} - 3 \text{ V})^2} = \frac{6 \times 10^{-3}}{25} \text{A/v}^2$   
=  $0.24 \times 10^{-3} \text{ A/V}^2$ 

For  $V_{GS} = 6$  V (between 3 and 8 V):

$$I_D = 0.24 \times 10^{-3} (6 \text{ V} - 3 \text{ V})^2 = 0.24 \times 10^{-3} (9)$$
  
= 2.16 mA



**Figure 40** Plotting the transfer curve for the MOSFET of Fig. 39.



as shown on Fig. 6.40. For  $V_{GS} = 10$  V (slightly greater than  $V_{GS(Th)}$ ):

$$I_D = 0.24 \times 10^{-3} (10 \text{ V} - 3 \text{ V})^2 = 0.24 \times 10^{-3} (49)$$
  
= 11.76 mA

as also appearing on Fig. 6.40. The four points are sufficient to plot the full curve for the range of interest as shown in Fig. 6.40.

For the Network Bias Line:

$$V_{GS} = V_{DD} - I_D R_D$$
  
= 12 V -  $I_D(2 \text{ k}\Omega)$   
Eq. (6.29):  $V_{GS} = V_{DD} = 12 \text{ V}|_{I_D} = 0 \text{ mA}$   
Eq. (6.30):  $I_D = \frac{V_{DD}}{R_D} = \frac{12 \text{ V}}{2 \text{ k}\Omega} = 6 \text{ mA}|_{V_{GS}} = 0 \text{ V}$ 

 $I_{D_Q} = 2.75 \text{ mA}$  $V_{GS_Q} = 6.4 \text{ V}$ 

The resulting bias line appears in Fig. 6.41. At the operating point:



and with





**Figure 42** Voltage-divider biasing arrangement for an *n*-channel enhancement MOSFET.

# **Voltage-Divider Biasing Arrangement**

A second popular biasing arrangement for the enhancement-type MOSFET appears in Fig. 6.42. The fact that  $I_G = 0$  mA results in the following equation for  $V_{GG}$  as derived from an application of the voltage-divider rule:

$$V_G = \frac{R_2 V_{DD}}{R_1 + R_2}$$
(31)

Applying Kirchhoff's voltage law around the indicated loop of Fig. 42 will result in

and

$$+V_G - V_{GS} - V_{R_S} = 0$$
$$V_{GS} = V_G - V_{R_S}$$

or

 $V_{GS} = V_G - I_D R_S \tag{32}$ 

For the output section:

and

or

$$V_{DS} = V_{DD} - V_{R_S} - V_{R_D}$$

$$V_{DS} = V_{DD} - I_D(R_S + R_D)$$
(33)

Since the characteristics are a plot of  $I_D$  versus  $V_{GS}$  and Eq. (6.32) relates the same two variables, the two curves can be plotted on the same graph and a solution determined at their intersection. Once  $I_{DQ}$  and  $V_{GSQ}$  are known, all the remaining quantities of the network such as  $V_{DS}$ ,  $V_D$ , and  $V_S$  can be determined.

 $V_{R_{\rm D}} + V_{DS} + V_{R_{\rm D}} - V_{DD} = 0$ 

Determine  $I_{DQ}$ ,  $V_{GSQ}$ , and  $V_{DS}$  for the network of Fig.43.



# Solution

Network:

Eq. (6.31): 
$$V_G = \frac{R_2 V_{DD}}{R_1 + R_2} = \frac{(18 \text{ M}\Omega)(40 \text{ V})}{22 \text{ M}\Omega + 18 \text{ M}\Omega} = 18 \text{ V}$$
  
Eq. (6.32):  $V_{GS} = V_G - I_D R_S = 18 \text{ V} - I_D (0.82 \text{ k}\Omega)$ 

When  $I_D = 0$  mA,

$$V_{GS} = 18 \text{ V} - (0 \text{ mA})(0.82 \text{ k}\Omega) = 18 \text{ V}$$

as appearing on Fig. 6.44. When  $V_{GS} = 0$  V,

$$V_{GS} = 18 \text{ V} - I_D(0.82 \text{ k}\Omega)$$
  
0 = 18 V - I\_D(0.82 k\Omega)  
$$I_D = \frac{18 \text{ V}}{0.82 \text{ k}\Omega} = 21.95 \text{ mA}$$

as appearing on Fig. 44.

EXAMPLE 12



Figure 44 Determining the *Q*-point for the network of Example 12.

**Device:** 

and

$$V_{GS(Th)} = 5 \text{ V}, \qquad I_{D(on)} = 3 \text{ mA with } V_{GS(on)} = 10 \text{ V}$$
  
Eq. (6.26):  $k = \frac{I_{D(on)}}{(V_{GS(on)} - V_{GS(Th)})^2}$   
 $= \frac{3 \text{ mA}}{(10 \text{ V} - 5 \text{ V})^2} = 0.12 \times 10^{-3} \text{ A/V}^2$   
 $I_D = k(V_{GS} - V_{GS(Th)})^2$   
 $= 0.12 \times 10^{-3}(V_{GS} - 5)^2$ 

which is plotted on the same graph (Fig. 6.44). From Fig. 6.44,

$$I_{DQ} \cong 6.7 \text{ mA}$$

$$V_{GSQ} = 12.5 \text{ V}$$
Eq. (6.33):  $V_{DS} = V_{DD} - I_D(R_S + R_D)$ 

$$= 40 \text{ V} - (6.7 \text{ mA})(0.82 \text{ k}\Omega + 3.0 \text{ k}\Omega)$$

$$= 40 \text{ V} - 25.6 \text{ V}$$

$$= 14.4 \text{ V}$$

# 6.7 SUMMARY TABLE

Now that the most popular biasing arrangements for the various FETs have been introduced, Table 6.1 reviews the basic results and demonstrates the similarity in approach for a number of configurations. It also reveals that the general analysis of dc configurations for FETs is not overly complex. Once the transfer characteristics are established, the network self-bias line can be drawn and the *Q*-point determined at the intersection of the device transfer characteristic and the network bias curve. The remaining analysis is simply an application of the basic laws of circuit analysis.

TABLE 6.1 FET Bias Configurations			
Туре	Configuration	Pertinent Equations	Graphical Solution
JFET Fixed-bias		$V_{GSQ} = -V_{GG}$ $V_{DS} = V_{DD} - I_D R_S$	$ \begin{array}{c c}  & I_D \\  & I_{DSS} \\ \hline  & V_P V_{GG} \mid 0 \\ \hline  & V_{GS} \end{array} $
JFET Self-bias	$\begin{bmatrix} V_{DD} \\ R_D \\ R_G \\ R_S \end{bmatrix}$	$V_{GS} = -I_D R_S$ $V_{DS} = V_{DD} - I_D (R_D + R_S)$	$Q-\text{point} \qquad \qquad$
JFET Voltage-divider bias	$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} R_b \\ R_c \\ R_s \end{bmatrix}$	$V_G = \frac{R_2 V_{DD}}{R_1 + R_2}$ $V_{GS} = V_G - I_D R_S$ $V_{DS} = V_{DD} - I_D (R_D + R_S)$	$\begin{array}{c} Q\text{-point} \\ \hline \\ V_P \\ \hline \\ V_P \\ \hline \\ V_G \\ V_G \\ V_G \\ V_{GS} \end{array}$
JFET Common-gate		$V_{GS} = V_{SS} - I_D R_S$ $V_{DS} = V_{DD} + V_{SS} - I_D (R_D + R_S)$	$\begin{array}{c} Q\text{-point} \\ \hline V_P \\ \hline V_{SS} \hline \hline V_{SS} \\ \hline V_{SS} \hline \hline V_{SS} \\ \hline V_{SS} \hline \hline V_{$
JFET $(V_{GSQ} = 0 \text{ V})$		$V_{GSQ} = 0 V$ $I_{DQ} = I_{DSS}$	$\begin{array}{c c} Q \text{-point} & I_D \\ I_{DSS} \\ V_{GSQ} = 0 \text{ V} \\ \hline V_P & 0 \\ V_{GS} \end{array}$
JFET $(R_D = 0 \ \Omega)$		$V_{GS} = -I_D R_S$ $V_D = V_{DD}$ $V_S = I_D R_S$ $V_{DS} = V_{DD} - I_S R_S$	$Q-\operatorname{point} V_{P}   V_{GS}   0 V_{GS}$
Depletion-type MOSFET Fixed-bias		$V_{GSQ} = +V_{GG}$ $V_{DS} = V_{DD} - I_D R_S$	$I_{DSS}$ $Q$ -point $V_P = 0$ $V_{GG} = V_{GS}$
Depletion-type MOSFET Voltage-divider bias		$V_G = \frac{R_2 V_{DD}}{R_1 + R_2}$ $V_{GS} = V_G - I_S R_S$ $V_{DS} = V_{DD} - I_D (R_D + R_S)$	$\begin{array}{c c} V_{G} & I_{D} \\ \hline R_{S} & Q \text{-point} \\ I_{DSS} & Q \\ \hline V_{P} & 0 & V_{G} & V_{GS} \end{array}$
Enhancement type MOSFET Feedback configuration		$V_{GS} = V_{DS}$ $V_{GS} = V_{DD} - I_D R_D$	$\begin{array}{c} V_{DD} & I_D \\ R_D \\ I_{D(on)} - & Q \text{-point} \\ \hline 0 & V_{GS(Th)} & V_{DD} & V_{GS} \end{array}$
Enhancement- type MOSFET Voltage-divider bias		$V_G = \frac{R_2 V_{DD}}{R_1 + R_2}$ $V_{GS} = V_G - I_D R_S$	$\begin{array}{c c} V_G & I_D \\ \hline R_S & Q\text{-point} \\ \hline 0 & V_{GS(\text{Th})} & V_G & V_{GS} \end{array}$

 $\rightarrow$ 

# Small-Signal Analysis

# **I.** FET SMALL-SIGNAL MODEL

The ac analysis of an FET configuration requires that a small-signal ac model for the FET be developed. A major component of the ac model will reflect the fact that an ac voltage applied to the input gate-to-source terminals will control the level of current from drain to source.

The gate-to-source voltage controls the drain-to-source (channel) current of an FET.

Recall from Chapter 6 that a dc gate-to-source voltage controlled the level of dc drain current through a relationship known as Shockley's equation:  $I_D = I_{DSS}(1 - V_{GS}/V_P)^2$ . The *change* in collector current that will result from a *change* in gate-to-source voltage can be determined using the transconductance factor  $g_m$  in the following manner:

$$\Delta I_D = g_m \, \Delta V_{GS} \tag{1}$$

The prefix *trans*- in the terminology applied to  $g_m$  reveals that it establishes a relationship between an output and input quantity. The root word *conductance* was chosen because  $g_m$  is determined by a voltage-to-current ratio similar to the ratio that defines the conductance of a resistor G = 1/R = I/V.

Solving for  $g_m$  in Eq. (1), we have:

$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} \tag{2}$$

# Graphical Determination of g<sub>m</sub>

If we now examine the transfer characteristics of Fig. 9.1, we find that  $g_m$  is actually the slope of the characteristics at the point of operation. That is,

$$g_m = m = \frac{\Delta y}{\Delta x} = \frac{\Delta I_D}{\Delta V_{GS}}$$
(3)

Following the curvature of the transfer characteristics, it is reasonably clear that the slope and, therefore,  $g_m$  increase as we progress from  $V_P$  to  $I_{DSS}$ . Or, in other words, as  $V_{GS}$  approaches 0 V, the magnitude of  $g_m$  increases.



Equation (2) reveals that  $g_m$  can be determined at any Q-point on the transfer characteristics by simply choosing a finite increment in  $V_{GS}$  (or in  $I_D$ ) about the Q-point and then finding the corresponding change in  $I_D$  (or  $V_{GS}$ , respectively). The resulting changes in each quantity are then substituted in Eq. (2) to determine  $g_m$ .

Determine the magnitude of  $g_m$  for a JFET with  $I_{DSS} = 8$  mA and  $V_P = -4$  V at the following dc bias points:

EXAMPLE.1

- (a)  $V_{GS} = -0.5$  V. (b)  $V_{GS} = -1.5$  V.
- (c)  $V_{GS} = -2.5$  V.

#### **Solution**

The transfer characteristics are generated as Fig. 2 using the procedure defined in Chapter 6. Each operating point is then identified and a tangent line is drawn at each point to best reflect the slope of the transfer curve in this region. An appropriate in-crement is then chosen for  $V_{GS}$  to reflect a variation to either side of each *Q*-point. Equation (9.2) is then applied to determine  $g_m$ .

(a) 
$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} \approx \frac{2.1 \text{ mA}}{0.6 \text{ V}} = 3.5 \text{ mS}$$

(b) 
$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} \approx \frac{1.8 \text{ mA}}{0.7 \text{ V}} \approx 2.57 \text{ mS}$$

(c) 
$$g_m = \frac{\Delta I_D}{\Delta V_{GS}} = \frac{1.5 \text{ mA}}{1.0 \text{ V}} = 1.5 \text{ mS}$$

Note the decrease in  $g_m$  as  $V_{GS}$  approaches  $V_P$ .



# Mathematical Definition of $g_m$

The graphical procedure just described is limited by the accuracy of the transfer plot and the care with which the changes in each quantity can be determined. Naturally, the larger the graph the better the accuracy, but this can then become a cumbersome problem. An alternative approach to determining  $g_m$  employs the approach used to find the ac resistance of a diode in Chapter 1, where it was stated that:

The derivative of a function at a point is equal to the slope of the tangent line drawn at that point.

If we therefore take the derivative of  $I_D$  with respect to  $V_{GS}$  (differential calculus) using Shockley's equation, an equation for  $g_m$  can be derived as follows:

$$g_{m} = \frac{\Delta I_{D}}{\Delta V_{GS}} \bigg|_{Q-\text{pt.}} = \frac{dI_{D}}{dV_{GS}} \bigg|_{Q-\text{pt.}} = \frac{d}{dV_{GS}} \bigg[ I_{DSS} \bigg( 1 - \frac{V_{GS}}{V_{P}} \bigg)^{2} \bigg]$$
  
$$= I_{DSS} \frac{d}{dV_{GS}} \bigg( 1 - \frac{V_{GS}}{V_{P}} \bigg)^{2} = 2I_{DSS} \bigg[ 1 - \frac{V_{GS}}{V_{P}} \bigg] \frac{d}{dV_{GS}} \bigg( 1 - \frac{V_{GS}}{V_{P}} \bigg)$$
  
$$= 2I_{DSS} \bigg[ 1 - \frac{V_{GS}}{V_{P}} \bigg] \bigg[ \frac{d}{dV_{GS}} (1) - \frac{1}{V_{P}} \frac{dV_{GS}}{dV_{GS}} \bigg] = 2I_{DSS} \bigg[ 1 - \frac{V_{GS}}{V_{P}} \bigg] \bigg[ 0 - \frac{1}{V_{P}} \bigg]$$
  
$$g_{m} = \frac{2I_{DSS}}{|V_{P}|} \bigg[ 1 - \frac{V_{GS}}{V_{P}} \bigg]$$
(4)

where  $|V_P|$  denotes magnitude only to ensure a positive value for  $g_m$ .

It was mentioned earlier that the slope of the transfer curve is a maximum at  $V_{GS} = 0$  V. Plugging in  $V_{GS} = 0$  V into Eq. (4) will result in the following equation for the maximum value of  $g_m$  for a JFET in which  $I_{DSS}$  and  $V_P$  have been specified:

$$g_m = \frac{2I_{DSS}}{|V_P|} \left[ 1 - \frac{0}{V_P} \right]$$

$$g_{m0} = \frac{2I_{DSS}}{|V_P|}$$
(5)

and

and

where the added subscript 0 reminds us that it is the value of  $g_m$  when  $V_{GS} = 0$  V. Equation (9.4) then becomes

$$g_m = g_{m0} \left[ 1 - \frac{V_{GS}}{V_P} \right] \tag{6}$$

#### **EXAMPLE.2**

For the JFET having the transfer characteristics of Example 1:

(a) Find the maximum value of  $g_m$ .

(b) Find the value of  $g_m$  at each operating point of Example 9.1 using Eq. (6) and compare with the graphical results.

#### **Solution**

(a) 
$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8 \text{ mA})}{4 \text{ V}} = 4 \text{ mS}$$
 (maximum possible value of  $g_m$ )

(b) At 
$$V_{GS} = -0.5$$
 V,

$$g_m = g_{m0} \left[ 1 - \frac{V_{GS}}{V_P} \right] = 4 \text{ mS} \left[ 1 - \frac{-0.5 \text{ V}}{-4 \text{ V}} \right] = 3.5 \text{ mS} \quad \text{(versus 3.5 mS} \text{ graphically)}$$

At 
$$V_{GS} = -1.5 \text{ V}$$
,  
 $g_m = g_{m0} \left[ 1 - \frac{V_{GS}}{V_P} \right] = 4 \text{ mS} \left[ 1 - \frac{-1.5 \text{ V}}{-4 \text{ V}} \right] = 2.5 \text{ mS}$  (versus 2.57 mS graphically)  
At  $V_{GS} = -2.5 \text{ V}$ ,  
 $g_m = g_{m0} \left[ 1 - \frac{V_{GS}}{V_P} \right] = 4 \text{ mS} \left[ 1 - \frac{-2.5 \text{ V}}{-4 \text{ V}} \right] = 1.5 \text{ mS}$  (versus 1.5 mS graphically)

The results of Example 9.2 are certainly sufficiently close to validate Eq. (4) through (6) for future use when  $g_m$  is required.

On specification sheets,  $g_m$  is provided as  $y_{fs}$  where y indicates it is part of an admittance equivalent circuit. The f signifies forward transfer parameter, and the s reveals that it is connected to the source terminal.

In equation form,

$$g_m = y_{fs} \tag{7}$$

# Plotting $g_m$ vs. $V_{GS}$

Since the factor  $\left(1 - \frac{V_{GS}}{V_P}\right)$  of Eq. (9.6) is less than 1 for any value of  $V_{GS}$  other than

0 V, the magnitude of  $g_m$  will decrease as  $V_{GS}$  approaches  $V_P$  and the ratio  $\frac{V_{GS}}{V_P}$  increases in magnitude. At  $V_{GS} = V_P$ ,  $g_m = g_{m0}(1 - 1) = 0$ . Equation (9.6) defines a straight line with a minimum value of 0 and a maximum value of  $g_m$  as shown by the plot of Fig. 9.3.



Figure 9.3 also reveals that when  $V_{GS}$  is one-half the pinch-off value,  $g_m$  will be one-half the maximum value.

Plot  $g_m$  vs.  $V_{GS}$  for the JFET of examples 1 and 2.

#### **Solution**



# Impact of $I_D$ on $g_m$

A mathematical relationship between  $g_m$  and the dc bias current  $I_D$  can be derived by noting that Shockley's equation can be written in the following form:

$$1 - \frac{V_{GS}}{V_P} = \sqrt{\frac{I_D}{I_{DSS}}} \tag{.8}$$

Substituting Eq. (9.8) into Eq. (9.6) will result in

$$g_m = g_{m0} \left( 1 - \frac{V_{GS}}{V_P} \right) = g_{m0} \sqrt{\frac{I_D}{I_{DSS}}}$$
 (9)

Using Eq. (9.9) to determine  $g_m$  for a few specific values of  $I_D$ , the results are

(a) If  $I_D = I_{DSS}$ ,

$$g_m = g_{m0} \sqrt{\frac{I_{DSS}}{I_{DSS}}} = g_{m0}$$

(b) If  $I_D = I_{DSS}/2$ ,

$$g_m = g_{m0} \sqrt{\frac{I_{DSS}/2}{I_{DSS}}} = 0.707 g_{m0}$$
(c) If  $I_D = I_{DSS}/4$ ,  
 $g_m = g_{m0} \sqrt{\frac{I_{DSS}/4}{I_{DSS}}} = \frac{g_{m0}}{2} = 0.5 g_{m0}$ 

EXAMPLE .4

Plot  $g_m$  vs.  $I_D$  for the JFET of Examples 1 through 3.

**Solution** 



The plots of Examples 9.3 and 9.4 clearly reveal that the highest values of  $g_m$  are obtained when  $V_{GS}$  approaches 0 V and  $I_D$  its maximum value of  $I_{DSS}$ .

# FET Input Impedance $Z_i$

The input impedance of all commercially available FETs is sufficiently large to assume that the input terminals approximate an open circuit. In equation form,

$$Z_i (FET) = \infty$$
(10)

For a JFET a practical value of 109  $\Omega$  (1000 M $\Omega$ ) is typical, while a value of 10<sup>15</sup>  $\Omega$  is typical for MOSFETs.

# FET Output Impedance $Z_o$

The output impedance of FETs is similar in magnitude to that of conventional BJTs. On FET specification sheets, the output impedance will typically appear as  $y_{os}$  with the units of  $\mu$ S. The parameter  $y_{os}$  is a component of an *admittance equivalent circuit*, with the subscript *o* signifying an *o*utput network parameter and *s* the terminal (source) to which it is attached in the model. For the JFET of Fig. 5.18,  $y_{os}$  has a range of 10 to 50  $\mu$ S or 20 k $\Omega$  ( $R = 1/G = 1/50 \ \mu$ S) to 100 k $\Omega$  ( $R = 1/G = 1/10 \ \mu$ S).

In equation form,

$$Z_o (\text{FET}) = r_d = \frac{1}{y_{os}} \tag{11}$$

The output impedance is defined on the characteristics of Fig. 9.6 as the slope of the horizontal characteristic curve at the point of operation. The more horizontal the curve, the greater the output impedance. If perfectly horizontal, the ideal situation is on hand with the output impedance being infinite (an open circuit)—an often applied approximation.

In equation form,

$$r_d = \frac{\Delta V_{DS}}{\Delta I_D} \bigg|_{V_{GS} = \text{constant}}$$
<sup>12</sup>)



**Figure .6** Definition of  $r_d$  using FET drain characteristics.

EXAMPLE 5

Determine the output impedance for the FET of Fig..7 for  $V_{GS} = 0$  V and  $V_{GS} = -2$  V at  $V_{DS} = 8$  V.



**Figure 7** Drain characteristics used to calculate  $r_d$  in Example.5.

### **Solution**

For  $V_{GS} = 0$  V, a tangent line is drawn and  $\Delta V_{DS}$  is chosen as 5 V, resulting in a  $\Delta I_D$  of 0.2 mA. Substituting into Eq. (12),

$$r_d = \frac{\Delta V_{DS}}{\Delta I_D} \bigg|_{V_{GS} = 0 \text{ V}} = \frac{5 \text{ V}}{0.2 \text{ mA}} = 25 \text{ k}\Omega$$

For  $V_{GS} = -2$  V, a tangent line is drawn and  $\Delta V_{DS}$  is chosen as 8 V, resulting in a  $\Delta I_D$  of 0.1 mA. Substituting into Eq. (12),

 $g_m$ 

$$r_d = \frac{\Delta V_{DS}}{\Delta I_D} \bigg|_{V_{GS} = -2 \text{ V}} = \frac{8 \text{ V}}{0.1 \text{ mA}} = 80 \text{ k}\Omega$$

revealing that  $r_d$  does change from one operating region to another, with lower values typically occurring at lower levels of  $V_{GS}$  (closer to 0 V).

### FET AC Equivalent Circuit

Now that the important parameters of an ac equivalent circuit have been introduced and discussed, a model for the FET transistor in the ac domain can be constructed. The control of  $I_d$  by  $V_{gs}$  is included as a current source  $g_m V_{gs}$  connected from drain to source as shown in Fig. 8. The current source has its arrow pointing from drain source to establish a 180° phase shift between output and input voltages as willoccur in actual operation.





The input impedance is represented by the open circuit at the input terminals and the output impedance by the resistor  $r_d$  from drain to source. Note that the gate to source voltage is now represented by  $V_{gs}$  (lower-case subscripts) to distinguish it from dc levels. In addition, take note of the fact that the source is common to both input and output circuits while the gate and drain terminals are only in "touch" through the controlled current source  $g_m V_{gs}$ .

In situations where  $r_d$  is ignored (assumed sufficiently large to other elements of the network to be approximated by an open circuit), the equivalent circuit is simply a current source whose magnitude is controlled by the signal  $V_{gs}$  and parameter  $g_m$ —clearly a voltage-controlled device.

Given 
$$y_{fs} = 3.8$$
 mS and  $y_{os} = 20 \mu$ S, sketch the FET ac equivalent model.

**EXAMPLE.6** 

#### **Solution**

$$g_m = y_{fs} = 3.8 \text{ mS}$$
 and  $r_d = \frac{1}{y_{os}} = \frac{1}{20 \ \mu\text{S}} = 50 \text{ k}\Omega$ 

resulting in the ac equivalent model of Fig. 9.9.



# **II.** JFET FIXED-BIAS CONFIGURATION

Now that the FET equivalent circuit has been defined, a number of fundamental FET small-signal configurations will be investigated. The approach will parallel the ac analysis of BJT amplifiers with a determination of the important parameters of  $Z_i$ ,  $Z_o$ , and  $A_v$  for each configuration.

The *fixed-bias* configuration of Fig. 9.10 includes the coupling capacitors  $C_1$  and  $C_2$  that isolate the dc biasing arrangement from the applied signal and load; they act as short-circuit equivalents for the ac analysis.





Once the level of  $g_m$  and  $r_d$  are determined from the dc biasing arrangement, specification sheet, or characteristics, the ac equivalent model can be substituted between the appropriate terminals as shown in Fig. 9.11. Note that both capacitors have the short-circuit equivalent because the reactance  $X_C = 1/(2\pi fC)$  is sufficiently small compared to other impedance levels of the network, and the dc batteries  $V_{GG}$  and  $V_{DD}$  are set to zero volts by a short-circuit equivalent.



Figure .11 Substituting the JFET ac equivalent circuit unit into the network of Fig. 9.10.

The network of Fig. is then carefully redrawn as shown in Fig. Note the defined polarity of  $V_{gs}$ , which defines the direction of  $g_m V_{gs}$ . If  $V_{gs}$  is negative, the direction of the current source reverses. The applied signal is and the output signal across  $R_D$  by  $V_o$ . represented by  $V_i$  $Z_i$ : Figure 12 clearly reveals that

$$Z_i = R_G \tag{13}$$

because of the open-circuit equivalence at the input terminals of the JFET.



Figure.12 Redrawn network of Fig. .11.

**Z<sub>o</sub>:** Setting  $V_i = 0$  V as required by the definition of  $Z_o$  will establish  $V_{gs}$  as 0V also. The result is  $g_m V_{gs} = 0$  mA, and the current source can be replaced by an open-circuit equivalent as shown in Fig. 13. The output impedance is

$$Z_o = R_D \| r_d \tag{14}$$

If the resistance  $r_d$  is sufficiently large (at least 10:1) compared to  $R_D$ , the approximation  $r_d || R_D \cong R_D$  can often be applied and

$$Z_o \cong R_D \qquad 15)$$



 $A_v$ : Solving for  $V_o$  in Fig. 12, we find

but

$$V_o = -g_m V_{gs}(r_d || R_D)$$
$$V_{gs} = V_i$$
$$V_o = -g_m V_i(r_d || R_D)$$

and

so that

$$A_{v} = \frac{V_{o}}{V_{i}} = -g_{m}(r_{d} || R_{D})$$
(16)

If  $r_d \ge 10R_D$ :

$$A_{v} = \frac{V_{o}}{V_{i}} = -g_{m}R_{D}$$

$$I_{d} \ge 10R_{D}$$

$$(17)$$

**Phase Relationship:** The negative sign in the resulting equation for  $A_v$  clearly reveals a phase shift of 180° between input and output voltages.

# EXAMPLE .7

The fixed-bias configuration of Example 1 had an operating point defined by  $V_{GS_Q} = -2$  V and  $I_{D_Q} = 5.625$  mA, with  $I_{DSS} = 10$  mA and  $V_P = -8$  V. The network is redrawn as Fig. 4 with an applied signal  $V_i$ . The value of  $y_{os}$  is provided as 40  $\mu$ S.(a) Determine  $g_m$ .

- (b) Find  $r_d$ .
- (c) Determine  $Z_i$ .
- (d) Calculate  $Z_o$ .
- (e) Determine the voltage gain  $A_{\nu}$ .
- (f) Determine  $A_v$  ignoring the effects of  $r_d$ .





#### **Solution**

(a) 
$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(10 \text{ mA})}{8 \text{ V}} = 2.5 \text{ mS}$$
  
 $g_m = g_{m0} \left( 1 - \frac{V_{GS_o}}{V_P} \right) = 2.5 \text{ mS} \left( 1 - \frac{(-2 \text{ V})}{(-8 \text{ V})} \right) = 1.88 \text{ mS}$   
(b)  $r_d = \frac{1}{y_{os}} = \frac{1}{40 \ \mu\text{S}} = 25 \text{ k}\Omega$ 

(c) 
$$Z_i = R_G = 1 \text{ M}\Omega$$

- (d)  $Z_o = R_D \| r_d = 2 \text{ k}\Omega \| 25 \text{ k}\Omega = 1.85 \text{ k}\Omega$
- (e)  $A_v = -g_m(R_D || r_d) = -(1.88 \text{ mS})(1.85 \text{ k}\Omega)$ = -3.48
- (f)  $A_v = -g_m R_D = -(1.88 \text{ mS})(2 \text{ k}\Omega) = -3.76$ As demonstrated in part (f), a ratio of 25 k $\Omega$ : 2 k $\Omega$  = 12.5:1 between  $r_d$  and  $R_D$  resulted in a difference of 8% in solution.

# **III.** JFET SELF-BIAS CONFIGURATION

# Bypassed R<sub>S</sub>

The fixed-bias configuration has the distinct disadvantage of requiring two dc volt-age sources. The *self-bias* configuration of Fig. 15 requires only one dc supply toestablish the desired operating point.



re 15 Self-bias JFET configuration.

The capacitor  $C_S$  across the source resistance assumes its short-circuit equivalence for dc, allowing  $R_S$  to define the operating point. Under ac conditions, the capacitor assumes the short-circuit state and "short circuits" the effects of  $R_S$ . If left in the ac, gain will be reduced as will be shown in the paragraphs to follow.

The JFET equivalent circuit is established in Fig. 16 and carefully redrawn in Fig. 17.



Figure 16 Network of Fig. 9.15 following the substitution of the JFET ac equivalent circuit.



Figure 17 Redrawn network of Fig. 16.

Since the resulting configuration is the same as appearing in Fig. 9.12, the resulting equations  $Z_i$ ,  $Z_o$ , and  $A_v$  will be the same.

Zi:

$$Z_i = R_G \tag{18}$$

$$Z_o$$
:

$$Z_o = r_d || R_D \tag{19}$$

If  $r_d \ge 10R_D$ ,

$$A_{\nu} = -g_m(r_d \| R_D) \tag{21}$$

If  $r_d \ 10R_D$ ,

Av:

$$A_{v} = -g_{m}R_{D} \qquad (22)$$

**Phase relationship:** The negative sign in the solutions for  $A_v$  again indicates a phase shift of 180° between  $V_i$  and  $V_o$ .

## Unbypassed R<sub>S</sub>

If  $C_s$  is removed from Fig 15, the resistor  $R_s$  will be part of the ac equivalent cir-cuit as shown in Fig..18. In this case, there is no obvious way to reduce the net-work to lower its level of complexity. In determining the levels of  $Z_i$ ,  $Z_o$ , and  $A_v$ , one must simply be very careful with notation and defined polarities and direction. Ini-tially, the resistance  $r_d$  will be left out of the analysis to form a basis for comparison.



**Figure 18** Self-bias JFET configuration including the effects of  $R_s$  with  $r_d = \infty \Omega$ .

 $Z_i$ : Due to the open-circuit condition between the gate and output network, the input remains the following:

$$Z_i = R_G \tag{23}$$

 $Z_o$ : The output impedance is defined by

$$Z_o = \frac{V_o}{I_o} \bigg|_{V_i = 0}$$

Setting  $V_i = 0$  V in Fig. 18 will result in the gate terminal being at ground poten-tial (0 V). The voltage across  $R_G$  is then 0 V, and  $R_G$  has been effectively "shortedout" of the picture.

with

so that

or

and

$$I_{o} + I_{D} = -g_{m} (I_{o} + I_{D})R_{S} = -g_{m}I_{o}R_{S} - g_{m}I_{D}R_{S}$$
$$I_{o}[1 + g_{m}R_{S}] = -I_{D}[1 + g_{m}R_{S}]$$

$$I_o[1 + g_m K_S] = -I_D[1 + g_m K_S]$$

$$I_o = -I_D \qquad \text{(the controlled current source } g_m V_{gs} = 0 \text{ A}$$
for the applied conditions)

Since

$$V_o = -I_D R_D$$

 $I_o + I_D = g_m V_{gS}$  $V_{gs} = -(I_o + I_D)R_S$ 

then

$$V_o = -(-I_o)R_D = I_o R_D$$

$$Z_o = \frac{V_o}{I_o} = R_D$$

$$r_d = \propto \Omega$$
(24)

Including the

and

If  $r_d$  is included in the network, the equivalent will appear as shown in Fig. 19.



we should try to find an expression for  $I_o$  in terms of  $I_D$ . Applying Kirchhoff's current law:

but

$$I_o = g_m V_{gs} + I_{r_d} - I_D$$
$$V_{r_d} = V_o + V_{gs}$$

 $I_o = g_m V_{gs} + \frac{V_o + V_{gs}}{r_d} - I_D$ and

$$I_o = \left(g_m + \frac{1}{r_d}\right) V_{gs} - \frac{I_D R_D}{r_d} - I_D \text{ using } V_o = -I_D R_D$$

Now,

so that

or

or

$$V_{gs} = -(I_D + I_o) R_S$$

$$I_{o} = -\left(g_{m} + \frac{1}{r_{d}}\right)(I_{D} + I_{o}) R_{S} - \frac{I_{D}R_{D}}{r_{d}} - I_{D}$$

with the result that 
$$I_o \left[ 1 + g_m R_s + \frac{R_s}{r_d} \right] = -I_o \left[ 1 + g_m R_s + \frac{R_s}{r_d} + \frac{R_D}{r_d} \right]$$
  
or 
$$I_o = \frac{-I_o \left[ 1 + g_m R_s + \frac{R_s}{r_d} + \frac{R_D}{r_d} \right]}{1 + g_m R_s + \frac{R_s}{r_d}}$$

$$g_m$$

and

 $\cong 1$ 

and 
$$Z_{o} = \frac{V_{o}}{I_{o}} = \frac{-I_{D}K_{D}}{-I_{D}\left(1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}} + \frac{R_{D}}{r_{d}}\right)}$$

$$1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}}$$
and finally,
$$Z_{o} = \frac{\left[1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}}\right]}{\left[1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}} + \frac{R_{D}}{r_{d}}\right]}R_{D}}$$

$$For r_{d} \ge 10 R_{D}, \left(1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}}\right) >> \frac{R_{D}}{r_{d}} \text{ and } 1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}} + \frac{R_{D}}{r_{d}}$$

$$\cong 1 + g_{m}R_{s} + \frac{R_{s}}{r_{d}} \text{ and }$$

$$(25a)$$

1 0

$$Z_o = R_D \qquad (25b)$$

 $A_v$ : For the network of Fig. 9.19, an application of Kirchhoff's voltage law on the input circuit will result in

$$V_i - V_{gs} - V_{R_s} = 0$$
$$V_{gs} = V_i - I_D R_s$$

The voltage across  $r_d$  using Kirchhoff's voltage law is

$$V_o - V_{R_s}$$

$$I' = \frac{V_o - V_{R_s}}{r_d}$$

. .

so that an application of Kirchhoff's current law will result in

$$I_D = g_m V_{gs} + \frac{V_o - V_{R_s}}{r_d}$$

Substituting for  $V_{gs}$  from above and substituting for  $V_o$  and  $V_{R_s}$  we have

$$I_{D} = g_{m}[V_{i} - I_{D}R_{S}] + \frac{(-I_{D}R_{D}) - (I_{D}R_{S})}{r_{d}}$$
$$I_{D}\left[1 + g_{m}R_{S} + \frac{R_{D} + R_{S}}{r_{d}}\right] = g_{m}V_{i}$$
$$I_{D} = \frac{g_{m}V_{i}}{1 + g_{m}R_{S} + \frac{R_{D} + R_{S}}{r_{d}}}$$

or

so that

and

The output voltage is then

$$V_{o} = -I_{D}R_{D} = -\frac{g_{m}R_{D}V_{i}}{1 + g_{m}R_{S} + \frac{R_{D} + R_{S}}{r_{d}}}$$

 $A_{v} = \frac{Vg_{m}R_{D}}{V_{i}} = \frac{1 + g_{m}R_{S}}{1 + g_{m}R_{S} + \frac{R_{D} + R_{S}}{r_{d}}}$ (26)

and

Again, if  $r_d \ge 10(R_D + R_S)$ ,

$$A_{\nu} = \frac{V_o}{V_i} = -\frac{g_m R_D}{1 + g_m R_S}$$

$$r_d \ge 10(R_D + R_S)$$
(27)

**Phase Relationship:** The negative sign in Eq. (26) again reveals that a  $180^{\circ}$  phase shift will exist between  $V_i$  and  $V_o$ .

The self-bias configuration of Example 6.2 has an operating point defined by  $V_{GS_Q} = -2.6$  V and  $I_{D_Q} = 2.6$  mA, with  $I_{DSS} = 8$  mA and  $V_P = -6$  V. The network is re-drawn as Fig. 20 with an applied signal  $V_i$ . The value of  $y_{os}$  is given as 20  $\mu$ S. (a) Determine  $g_m$ .

(b) Find  $r_d$ .

(c) Find  $Z_i$ .

- (d) Calculate  $Z_o$  with and without the effects of  $r_d$ . Compare the results.
- (e) Calculate  $A_v$  with and without the effects of  $r_d$ . Compare the results.



Figure 20 Network for Example 9.8.

**Solution** 

(a) 
$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(8 \text{ mA})}{6 \text{ V}} = 2.67 \text{ mS}$$
  
 $g_m = g_{m0} \left( 1 - \frac{V_{GS_o}}{V_P} \right) = 2.67 \text{ mS} \left( 1 - \frac{(-2.6 \text{ V})}{(-6 \text{ V})} \right) = 1.51 \text{ mS}$   
(b)  $r_d = \frac{1}{y_{os}} = \frac{1}{20 \ \mu\text{S}} = 50 \text{ k}\Omega$ 

- (c)  $Z_i = R_G = 1 \text{ M}\Omega$
- (d) With  $r_d$ :

$$r_d = 50 \text{ k}\Omega > 10 R_D = 33 \text{ k}\Omega$$

Therefore,

$$Z_o = R_D = 3.3 \text{ k}\Omega$$

If  $r_d = \infty \Omega$ 

$$Z_o = R_D = 3.3 \text{ k}\Omega$$

(e) With  $r_d$ :

$$A_{v} = \frac{-g_{m}R_{D}}{1 + g_{m}R_{S} + \frac{R_{D} + R_{S}}{r_{d}}} = \frac{-(1.51 \text{ mS})(3.3 \text{ k}\Omega)}{1 + (1.51 \text{ mS})(1 \text{ k}\Omega) + \frac{3.3 \text{ k}\Omega + 1 \text{ k}\Omega}{50 \text{ k}\Omega}}$$
$$= -1.92$$

EXAMPLE 8

Without  $r_d$ :

$$A_{v} = \frac{-g_{m} R_{D}}{1 + g_{m} R_{S}} = \frac{-(1.51 \text{ mS})(3.3 \text{ k}\Omega)}{1 + (1.51 \text{ mS})(1 \text{ k}\Omega)} = -1.98$$

As above, the effect of  $r_d$  was minimal because the condition  $r_d \ge 10(R_D + R_S)$  was satisfied.

Note also that the typical gain of a JFET amplifier is less than that generally encountered for BJTs of similar configurations. Keep in mind, however, that  $Z_i$  is magnitudes greater than the typical  $Z_i$  of a BJT, which will have a very positive effect on the overall gain of a system.

# **IV.** VOLTAGE-DIVIDERCONFIGURATION

The popular voltage-divider configuration for BJTs can also be applied to JFETs as demonstrated in Fig. 9.21.



JFET voltage-divider configuration.

Substituting the ac equivalent model for the JFET will result in the configuration of Fig. 22. Replacing the dc supply  $V_{DD}$  by a short-circuit equivalent has grounded one end of  $R_1$  and  $R_D$ . Since each network has a common ground,  $R_1$  can be brought down in parallel with  $R_2$  as shown in Fig. 23.  $R_D$  can also be brought down to ground but in the output circuit across  $r_d$ . The resulting ac equivalent network now has the basic format of some of the networks already analyzed.



Figure.22 Network of Fig. 21 under ac conditions.



 $Z_i$ :  $R_1$  and  $R_2$  are in parallel with the open-circuit equivalence of the JFET resulting in

$$Z_i = R_1 || R_2 \tag{28}$$

 $Z_o$ : Setting  $V_i = 0$  V will set  $V_{gs}$  and  $g_m V_{gs}$  to zero and

$$Z_o = r_d \| R_D \tag{29}$$

For  $r_d$  10 $R_D$ ,

$$Z_o \cong R_D \qquad (30)$$

 $A_{v}$ :

and 
$$V_o = -g_m V_{gs}(r_d \| R_D)$$

so that

$$A_{v} = \frac{V_{o}}{V_{i}} = \frac{-g_{m}V_{gs}(r_{d}||R_{D})}{V_{gs}}$$

 $V_{gs} = V_i$ 

$$A_v = \frac{V_o}{V_i} = -g_m(r_d || R_D)$$
(31)

and

If  $r_d \ 10R_D$ ,

 $A_{\nu} = \frac{V_o}{V_i} \cong -g_m R_D$   $r_d \ge 10 R_D$ (32)

Note that the equations for  $Z_o$  and  $A_v$  are the same as obtained for the fixed-bias and self-bias (with bypassed  $R_s$ ) configurations. The only difference is the equation for  $Z_i$ , which is now sensitive to the parallel combination of  $R_1$  and  $R_2$ .

# **V.JFET SOURCE-FOLLOWER(COMMON-DRAIN) CONFIGURATION**

The JFET equivalent of the BJT emitter-follower configuration is the source-follower configuration of Fig. 9.24. Note that the output is taken off the source terminal and, when the dc supply is replaced by its short-circuit equivalent, the drain is grounded (hence, the terminology common-drain).



Substituting the JFET equivalent circuit will result in the configuration of Fig. 25. The controlled source and internal output impedance of the JFET are tied to ground at one end and  $R_s$  on the other, with  $V_o$  across  $R_s$ . Since  $g_m V_{gs}$ ,  $r_d$ , and  $R_s$  are connected to the same terminal and ground, they can all be placed in parallel as shown in Fig. 26. The current source reversed direction but  $V_{gs}$  is still defined between the gate and source terminals.



Figure 25 Network of Fig. 24 following the substitution of the JFET ac equivalent model.

Figure.26 Network of Fig. 25 redrawn.

# $Z_i$ : Figure 9.26 clearly reveals that $Z_i$ is defined by

$$Z_i = R_G \tag{33}$$

 $Z_o$ : Setting  $V_i = 0$  V will result in the gate terminal being connected directly to ground as shown in Fig. 9.27. The fact that  $V_{gs}$  and  $V_o$  are across the same parallel network results in  $V_o = -V_{gs}$ .



Applying Kirchhoff's current law at node s,

 $I_o$ 

$$I_o + g_m V_{gs} = I_{r_d} + I_{R_s}$$
$$= \frac{V_o}{r_d} + \frac{V_o}{R_s}$$
$$= V_o \left[ \frac{1}{r_d} + \frac{1}{R_s} \right] - g_m V_{gs}$$
$$= V_o \left[ \frac{1}{r_d} + \frac{1}{R_s} \right] - g_m [-V_o]$$
$$= V_o \left[ \frac{1}{r_d} + \frac{1}{R_s} + g_m \right]$$

The result is

and 
$$Z_o = \frac{V_o}{I_o} = \frac{V_o}{V_o \left[\frac{1}{r_d} + \frac{1}{R_S} + g_m\right]} = \frac{1}{\frac{1}{r_d} + \frac{1}{R_S} + g_m} = \frac{1}{\frac{1}{r_d} + \frac{1}{R_S} + \frac{1}{1/g_m}}$$

which has the same format as the total resistance of three parallel resistors. Therefore,

$$Z_o = r_d \|R_S\| \ 1/g_m \tag{34}$$

For  $r_d \ge 10R_S$ ,

$$Z_o \cong R_S \| 1/g_m$$

$$_{r_d \ge 10R_S}$$
(35)

 $A_{\nu}$ : The output voltage  $V_o$  is determined by

$$V_o = g_m V_{gs}(r_d \| R_S)$$

and applying Kirchhoff's voltage law around the perimeter of the network of Fig. 9.26 will result in

$$V_i = V_{gs} + V_o$$

and

so that

$$V_{gs} - V_i - V_o$$
$$V_o = g_m (V_i - V_o) (r_d || R_S)$$

 $V_o = g_m V_i(r_d || R_S) - g_m V_o(r_d || R_S)$ 

or

$$V_{o}[1 + g_{m}(r_{d} || R_{S})] = g_{m}V_{i}(r_{d} || R_{S})$$
$$A_{v} = \frac{V_{o}}{V_{i}} = \frac{g_{m}(r_{d} || R_{S})}{1 + g_{m}(g_{d} || R_{S})}$$

so that

In the absence of  $r_d$  or if  $r_d \ge 10R_s$ ,

$$A_{\nu} = \frac{V_o}{V_i} \cong \frac{g_m R_S}{1 + g_m R_S}$$
(37)

Since the bottom of Eq. (36) is larger than the numerator by a factor of one, the gain can never be equal to or greater than one (as encountered for the emitter-fol-lower BJT network).

**Phase Relationship:** Since  $A_v$  of Eq. (36) is a positive quantity,  $V_o$  and  $V_i$  are in phase for the JFET source-follower configuration.

A dc analysis of the source-follower network of Fig. 9.28 will result in  $V_{GS_Q} = -2.86$  V and **EXAMPLE** .9  $I_{D_Q} = 4.56$  mA.

(a) Determining  $g_m$ .

(b) Find  $r_d$ .

- (c) Determine  $Z_i$ .
- (d) Calculate  $Z_o$  with and without  $r_d$ . Compare results.
- (e) Determine  $A_v$  with and without  $r_d$ . Compare results.



⊥0 V

(36)

Figure .28 Network to be analyzed in Example .9.

**Solution** 

(a) 
$$g_{m0} = \frac{2I_{DSS}}{|V_P|} = \frac{2(16 \text{ mA})}{4 \text{ V}} = 8 \text{ mS}$$
  
 $g_m = g_{m0} \left( 1 - \frac{V_{GS_0}}{V_P} \right) = 8 \text{ mS} \left( 1 - \frac{(-2.86 \text{ V})}{(-4 \text{ V})} \right) = 2.28 \text{ mS}$   
(b)  $r_d = \frac{1}{y_{os}} = \frac{1}{25 \mu \text{S}} = 40 \text{ k}\Omega$   
(c)  $Z_i = R_G = 1 \text{ M}\Omega$   
(d) With  $r_d$ :

$$Z_o = r_d \|R_s\| 1/g_m = 40 \text{ k}\Omega\| 2.2 \text{ k}\Omega\| 1/2.28 \text{ mS}$$
  
= 40 k\Omega \|2.2 k\Omega \| 438.6 \Omega   
= **362.52 \Omega**

revealing that  $Z_o$  is often relatively small and determined primarily by  $1/g_m$ . Without  $r_d$ :

$$Z_o = R_S ||1/g_m| = 2.2 \text{ k}\Omega ||438.6 \ \Omega = 365.69 \ \Omega$$

revealing that  $r_d$  typically has little impact on  $Z_o$ .

(e) With  $r_d$ :

$$A_{\nu} = \frac{g_m(r_d || R_S)}{1 + g_m(r_d || R_S)} = \frac{(2.28 \text{ mS})(40 \text{ k}\Omega || 2.2 \text{ k}\Omega)}{1 + (2.28 \text{ mS})(40 \text{ k}\Omega || 2.2 \text{ k}\Omega)}$$
$$\frac{(2.28 \text{ mS})(2.09 \text{ k}\Omega)}{1 + (2.28 \text{ mS})(2.09 \text{ k}\Omega)} = \frac{4.77}{1 + 4.77} = 0.83$$

which is less than 1 as predicted above. Without  $r_d$ :

$$A_{v} = \frac{g_{m}R_{S}}{1 + g_{m}R_{S}} = \frac{(2.28 \text{ mS})(2.2 \text{ k}\Omega)}{1 + (2.28 \text{ mS})(2.2 \text{ k}\Omega)}$$
$$= \frac{5.02}{1 + 5.02} = 0.83$$

revealing that  $r_d$  usually has little impact on the gain of the configuration.

# **VI.** JFET COMMON-GATECONFIGURATION

The last JFET configuration to be analyzed in detail is the common-gate configura-tion of Fig..29, which parallels the common-base configuration employed with BJT transistors.

Substituting the JFET equivalent circuit will result in Fig..30. Note the continuing requirement that the controlled source  $g_m V_{gs}$  be connected from drain to source with  $r_d$  in parallel. The isolation between input and output circuits has obviously been lost since the gate terminal is now connected to the common ground of the network. In addition, the resistor connected between input terminals is no longer  $R_G$  but the resistor  $R_S$  connected from source to ground. Note also the location of the control-ling voltage  $V_{gs}$  and the fact that it appears directly across the resistor  $R_S$ .



**Figure .29** JFET common-gate configuration.



**Figure 30** Network of Fig. 9.29 following substitution of JFET ac equivalent model.

(38)

 $Z_i$ : The resistor  $R_s$  is directly across the terminals defining  $Z_i$ . Let us therefore find the impedance  $Z_i$  of Fig. 29, which will simply be in parallel with  $R_s$  when  $Z_i$  is defined.

The network of interest is redrawn as Fig. 9.31. The voltage  $V' = -V_{gs}$ . Applying Kirchhoff's voltage law around the output perimeter of the network will result in



Applying Kirchhoff's current law at node a results in

and

$$I' = I_{r_d} - g_m V_{gs} = \frac{(V' - I'R_D)}{r_d} - g_m V_{gs}$$

 $I'\left[1 + \frac{R_D}{r_d}\right] = V'\left[\frac{1}{r_d} + g_m\right]$ 

 $I' + g_m V_{gs} = I_{r_d}$ 

or

$$I' = \frac{V'}{r_d} - \frac{I'R_D}{r_d} - g_m[-V']$$

so that

$$Z'_{i} = \frac{V'}{I'} = \frac{\left[1 + \frac{R_{D}}{r_{d}}\right]}{\left[g_{m} + \frac{1}{r_{d}}\right]}$$

and

or

$$Z'_i = \frac{V'}{I'} = \frac{r_d + R_D}{1 + g_m r_d}$$

and

$$Z_i = R_S \| Z_i'$$

results in

$$Z_i = R_S \left\| \left[ \frac{r_d + R_D}{1 + g_m r_d} \right] \right\|$$
(39)

If  $r_d$  10 $R_D$ , Eq. (38) permits the following approximation since  $R_D/r_d \ll 1$  and  $1/r_d \ll g_m$ :

$$Z_{i}^{\prime} = \frac{\left[1 + \frac{R_{D}}{r_{d}}\right]}{\left[g_{m} + \frac{1}{r_{d}}\right]} \approx \frac{1}{g_{m}}$$

$$Z_{i} \approx R_{S} \|1/g_{m}\|_{r_{d} \geq 10R_{D}}$$

$$(40)$$

and

**Z**<sub>o</sub>: Substituting  $V_i = 0$  V in Fig. .30 will "short-out" the effects of  $R_s$  and set  $V_{gs}$  to 0 V. The result is  $g_m V_{gs} = 0$ , and  $r_d$  will be in parallel with  $R_D$ . Therefore,

$$Z_o = R_D \| r_d \tag{41}$$

For  $r_d \ge 10R_D$ ,

$$Z_o \cong R_D \qquad (42)$$

 $A_{v}$ : Figure 30 reveals that

and

$$V_i = -V_{gs}$$
$$V_o = I_D R_D$$

The voltage across  $r_d$  is

and 
$$V_{r_d} = V_o - V_i$$
$$I_{r_d} = \frac{V_o - V_i}{r_d}$$

Applying Kirchhoff's current law at node b in Fig. 30 results in

and

$$I_{r_d} + I_D + g_m V_{gs} = 0$$

$$I_D = -I_{r_d} - g_m V_{gs}$$

$$= -\left[\frac{V_o - V_i}{r_d}\right] - g_m [-V_i]$$

$$I_D = \frac{V_i - V_o}{r_d} + g_m V_i$$

$$V_o = I_D R_D = \left[\frac{V_i - V_o}{r_d} + g_m V_i\right] R_D$$

$$= \frac{V_i R_D}{r_d} - \frac{V_o R_D}{r_d} + g_m$$

$$V_o \left[1 + \frac{R_D}{r_d}\right] = V_i \left[\frac{R_D}{r_d} + g_m R_D\right]$$

and

so that

with

$$A_{\nu} = \frac{V_o}{V_i} = \frac{\left[g_m R_D + \frac{R_D}{r_d}\right]}{\left[1 + \frac{R_D}{r_d}\right]}$$
(43)

For  $r_d \ 10R_D$ , the factor  $R_D/r_d$  of Eq. (43) can be dropped as a good approximation and

$$A_{\nu} = g_m R_D \qquad (44)$$

**Phase Relationship:** The fact that  $A_v$  is a positive number will result in an *in-phase* relationship between  $V_o$  and  $V_i$  for the common-gate configuration.

Although the network of Fig. 32 may not initially appear to be of the common-gate variety, a close examination will reveal that it has all the characteristics of Fig. 29. If  $V_{GS_o} = -2.2$  V and  $I_{D_o} = 2.03$  mA:

- (a) Determine  $g_m$ .
- (b) Find  $r_d$ .
- (c) Calculate  $Z_i$  with and without  $r_d$ . Compare results.
- (d) Find  $Z_o$  with and without  $r_d$ . Compare results.
- (e) Determine  $V_o$  with and without  $r_d$ . Compare results.



Figure 32

**Solution** 

(b) 
$$r_d = \frac{1}{y_{os}} = \frac{1}{50 \ \mu \text{S}} = 20 \ \text{kG}$$

(c) With  $r_d$ :

$$Z_{i} = R_{S} \left\| \left[ \frac{r_{d} + R_{D}}{1 + g_{m} r_{d}} \right] = 1.1 \text{ k}\Omega \right\| \left[ \frac{20 \text{ k}\Omega + 3.6 \text{ k}\Omega}{1 + (2.25 \text{ ms})(20 \text{ k}\Omega)} \right]$$
$$= 1.1 \text{ k}\Omega \| 0.51 \text{ k}\Omega = 0.35 \text{ k}\Omega$$

 $g_m$ 

**EXAMPLE 10** 

Without  $r_d$ :

$$Z_i = R_S ||1/g_m| = 1.1 \text{ k}\Omega ||1/2.25 \text{ ms} = 1.1 \text{ k}\Omega ||0.44 \text{ k}\Omega||$$
  
= 0.31 kΩ

Even though the condition,

$$r_d \ge 10R_D = > 20 \text{ k}\Omega \ge 10(3.6 \text{ k}\Omega) = > 20 \text{ k}\Omega \ge 36 \text{ k}\Omega$$

is *not* satisfied, both equations result in essentially the same level of impedance. In this case,  $1/g_m$  was the predominant factor.

(d) With  $r_d$ :

$$Z_o = R_D \| r_d = 3.6 \text{ k}\Omega \| 20 \text{ k}\Omega = 3.05 \text{ k}\Omega$$

Without  $r_d$ :

$$Z_o = R_D = 3.6 \text{ k}\Omega$$

Again the condition  $r_d \ge 10R_D$  is *not* satisfied, but both results are reasonably close.  $R_D$  is certainly the predominant factor in this example.

(e) With  $r_d$ :

$$A_{v} = \frac{\left[g_{m}R_{D} + \frac{R_{D}}{r_{d}}\right]}{\left[1 + \frac{R_{D}}{r_{d}}\right]} = \frac{\left[(2.25 \text{ mS})(3.6 \text{ k}\Omega) + \frac{3.6 \text{ k}\Omega}{20 \text{ k}\Omega}\right]}{\left[1 + \frac{3.6 \text{ k}\Omega}{20 \text{ k}\Omega}\right]}$$
$$= \frac{8.1 + 0.18}{1 + 0.18} = 7.02$$

and  $A_v = \frac{V_o}{V_i} = \blacktriangleright V_o = A_v V_i = (7.02)(40 \text{ mV}) = 280.8 \text{ mV}$ 

Without  $r_d$ :

with

$$A_v = g_m R_D = (2.25 \text{ mS})(3.6 \text{ k}\Omega) = 8.1$$
  
 $V_o = A_v V_i = (8.1)(40 \text{ mV}) = 324 \text{ mV}$ 

In this case, the difference is a little more noticeable but not dramatically so.

# **SUMMARY TABLE**



 $g_m$ 



# $g_m$